THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH
EUGENE R. SMITH MAURICE J. BABB

VOLUME V DECEMBER, 1912 NUMBER 2

EDUCATIONAL VALUES OF GEOMETRY.

BY F. F. DECKER.

(*Continued from September issue.*)

and the high school in saving the major portion of the time the boy must otherwise work as an apprentice on entering a trade. The tradesman, following, carried the audience with him when he said that such a system would sentence the boy to the trade of his father and with a great probability that it would be a life sentence, as a boy to get out of a trade or to improve conditions in that trade must be trained to think. He concluded with a plea that his son be trained as a thinker.

The writer has been asked to examine with reference to accepting it for college entrance a text written for vocational high schools in which parallel lines are defined as lines that have the same direction. From this definition the student is asked to deduce the following facts in regard to two points: first, that parallel lines, that is lines having the same direction, drawn through them will be everywhere equi-distant, and second, that lines so drawn will never meet. This is not an exceptional instance but the dominating plan of the book. Now what respect for a great system of deductive reasoning will this inspire? In what kind of thinking will this train the pupil? Teachers of geometry seem many times to forget that it is a distinct science from that of its sister, algebra. So much has been said about teaching them side by side—together, if you please—that one is at times tempted to forget that such a combination is, if we may borrow

a figure from natural science, a mechanical mixture rather than a chemical compound. Such a method has doubtless its advantages but it has a fundamental disadvantage in obscuring the development of the geometry.

Geometry teachers must be alert to the movements which from time to time seem to undermine the teaching of their science. They it is, who may gain inspiration from this contact with truth, with this science edited by Euclid, a master mind of his age, improved by grace of the keener discernment of the nineteenth century, not yet perfect, of course, for the world must have an intellectual standard, the successive polishings of which may serve as an indicator of the intellectual progress of the world.

The result is not in doubt. School authorities will yield to our demands who knows to what extent? We may be given the same amount of time as is now allotted to the laboratory sciences. For a large part of the student's time must be used in making deductions for himself under proper guidance just as in the laboratory science he must perform the experiment and make the observation and inference for himself. At first the deductions will be very simple. When he adopts the agreement, on historical grounds, that by space we mean among other things a place where not more than one line passing through a point can be parallel to a given line, he may deduce and state relations between three straight lines, two of which pass through a point, three straight lines, two of which are parallel to a third, and three straight lines, two of which are parallel. As the work advances he will usually need to be told what he is to deduce but by no means always.

In addition to a laboratory period the student will need a laboratory manual, a manual covering the whole laboratory course fully prepared in advance. For no teacher will hazard injuring the cause by getting the concession of laboratory periods without first having the course fully mapped out. This manual could be written by the teacher himself if he is a mathematically trained teacher of experience, and could then be given to his students in mimeographed form. Begin such a manual after the Christmas holidays and revise it as you watch your class progress. Then keep revising it. You will reap great benefit whether or not you ever have it published. Passing over the

various lists of sequences of propositions available, the writer knows of very few manuals; so that you may not find one to suit. The problem of what the manual should contain has been very little discussed, the question always taking the form, shall we teach with or without a book and no one believes in teaching geometry without some kind of a book. The problem must be solved by classroom experience and subsequent discussion, but always by teachers with the mathematical viewpoint; for practice alone will not make perfect. One principle would seem to be that where the teacher must handle more than say fourteen students with less than a certain minimum of time for the laboratory part of the course, more must be added to the manual. This is simply saying that the amount of geometric training the student is to get will be a function of the time and energy the teacher has at his disposal for the work.

The laboratory method seems to be the natural one for training the student to think. We are told by teachers who have tried it that it is practical without any modification of the present examination system. We must not be so carried away with it however as to forget another purpose for teaching geometry, the appreciation of geometry as a whole as a model of truth and beauty. In some ways the methods of procedure in striving to accomplish these two may be sharply contrasted. For the former purpose the dissection of the system into theorems and the determination of the best sequence of them must be carried out in the laboratory. The first deductions must be simple. The next theorems must be arranged somewhat according to their methods of proof, so that the student may early get his first lessons in methods of attack. The conclusions will not always be stated. The goal will almost invariably be the Q.E.D. On the other hand the geometry of the second type will be written by the specialist in logical foundations, and its arrangement will have regard primarily for the beautiful.

This second text will be much briefer than the first in the number of theorems and will have them arranged entirely with regard to the finished whole. Mathematical elegance will be exemplified by including in the definition only the essential properties of the thing defined and by using in the proof of the theorem only the essential properties necessary to establish the conclusion. The study of a necessary but sufficient set of pos-

tulates will be a real lesson in the exact adaptation of means to ends. In another sense there will be no ends and therefore no possibility of contempt for any parts as merely means. All will be equally important parts of the system. If there be lofty peaks, their grandeur will in no way obscure the charm of the rolling lowlands.

Here will be developed a thoroughgoing respect for consistency, for truth if you please, for consistency will be the test of advancement. Here will be the moral lesson of the absolute separation between truth and error, a separation not depending on the judgment of the individual. The student may lack the keen discernment of the teacher but the appreciation of real beauty and the reverence for ultimate truth are to some extent contagious. What then of the teacher who disregards the opportunity to lead the student into this immortal realm. How great the shame of the teacher who asks the pupil to take the trouble to prove a proposition in order that he may make some practical application.

The student who studies Shakespeare to appreciate him will get some benefit in theme writing while the one who studies him with a system of pigeonholes for fifty-seven varieties of adjective phrases may not fail to get something of Shakespeare's meaning. Similarly it is not contemplated that the advantages derived from these two methods of studying geometry will be sharply divided, some belonging to each. What we do say is that the two methods are complementary, each essential. Our present prevailing text book is a compromise between the two and not well adapted to either, although most teachers are trying to make them serve both purposes and with the geometrical spirit and painstaking attention to details are having, in spite of the book, a commendable measure of success.

If we are to have the course composed of two distinct parts, questions arise concerning the length of time to be devoted to each and the place of each in the curriculum. On the assurance of teachers with experience in handling the laboratory method we may state that that part of the course can be disposed of in a year with no more work on the part of the student than is devoted to the subject under the old system. Any statement of the time required for the second part, other than to suggest that it would not seem to be a lengthy matter, seems im-

possible. The six-and-six division of the primary-secondary school system may become a reality. If so would that not be an auspicious time to urge our claim to one and one half years for plane geometry if we need the extra time to teach the subject as it should be taught?

As to the place of each in the curriculum some may say that the second should come in the same year as the first, even if a slight limitation of the field were thus necessitated; others may say that it should come in the year following; others that it should come in the senior year where it may serve as a review for those about to enter college. There may be those who say that the place for such a course is in college. We do not know of any college giving such a course at the present time. If the high school will give the former only, they will, in spite of a lamentable omission, be taking, it seems to the writer, a step in advance. Then the colleges will find themselves compelled to give the latter.

In retrospect we have aimed to state that geometry should be taught because it is what it is—a deductive system; that the essential educational values of a training in geometry are a power to reason deductively and an appreciation of truth and beauty as exemplified by the science; and that the two may be more efficiently acquired by two separate treatments than by a compromise.

During the course of the remarks we have touched on many points that give an opportunity for a difference of opinion. In order to refresh your minds as to the particular matters concerning which, at some time or other during the progress of the paper, you were planning to favor us by way of discussion, we may recall some of these points. It may be claimed that geometry already is too geometrical, or that we have not selected the essentially geometrical values of a training in geometry—the vital reasons for teaching it—that the present methods of teaching the subject, in general use, are accomplishing these purposes most efficiently, or that some other method is better. Further there may be those agreeing in all these points with the writer who will volunteer an opinion as to the proper length and place of each of these parts of the course in the curriculum.

SYRACUSE UNIVERSITY,
SYRACUSE, N. Y.